Numerical Investigation of Thermal Performance in Cross Flow Around Square Array of Circular Cylinders
A. Jugal M. Panchal, B. A M Lakdawala

Abstract - Flow over square array of circular cylinders was studied for various Reynolds numbers i.e. 40, 50, 100, 150 and 200 by varying different Prandtl number i.e. 0.7, 1, 7, 10 and 20. Flow and Thermal analyses were done on square array of cylinders having length between centres of two cylinders to diameter ratio (L/D) 1.25. Effect of asymmetric arrangement of cylinders in domain and change of boundary conditions were also analysed. The analyses were done by solving two dimensional, unsteady incompressible Navier-stokes and energy equation using Fluent (version 6.2.16.). The effect of surface heating of cylinder, change in Reynolds number on wake formation and vortex generation, change in Reynolds number, change in symmetry of cylinders and change in Prandtl number on average Nusselt number was numerically studied. The validations was done for average Nusselt number on single cylinder at Re = 150 and Re = 200 and results were compared and found in good agreement with referred experimental and numerical work. It was found that qualitatively with increase in Prandtl number, temperature variation decreased around cylinders for constant value of Reynolds number and quantitatively, with increase in 5 unit & 10 unit asymmetry of cylinders, increase in average Nusselt no. were 4.69% and 7.17% respectively. Similarly average Nusselt number was 18 to 25% higher in case of Uniform Heat Flux (UHF) boundary Condition compare to Constant Wall Temperature (CWT) boundary condition for symmetric domain. The Correlation was also presented for average Nusselt number as function of Reynolds number and Prandtl number in case of symmetric and asymmetric domain of cylinder array.

Keywords: Correlation of average Nusselt number, CFD-FLUENT, Square array of circular cylinders, effect of asymmetry on average Nusselt number.

1. INTRODUCTION

Heat transfer is the most basic phenomena which happen in routine life. It can be classified as conduction, convection and radiation. Convection heat transfer is complicated by the fact that it involves fluid motion as well as heat conduction. The convection heat transfer strongly depends upon fluid properties such as thermal conductivity, density, fluid velocity, specific heat, as well as dynamic viscosity etc. due to viscosity of fluid, it is observed that the fluid layer in direct contact with a solid surface sticks to the surface and there is no slip. This phenomenon is known as the no-slip condition which is responsible for the development of the velocity profile for flow. A similar type of phenomenon occurs for the temperature as well. Due to these phenomena velocity and temperature gradients occurs. These gradients for velocity and temperature are zero at cylinder surface and maximum at some thickness \( \delta \) and \( \delta_\varepsilon \) from cylinder surface respectively. These thicknesses \( \delta \) and \( \delta_\varepsilon \) are known as velocity boundary layer thickness and thermal boundary layer thickness respectively. In case of forced heat transfer; to determine which thickness is larger among these two, the non-dimensional number called Prandtl Number (Pr) comes into picture. And the other dimensionless parameter Reynolds number is important in order to categorize flow over cylinder i.e. Laminar, transition or turbulent.

1.1 LITERATURE REVIEW

From the study of literature review, it was found that there are few papers available for heat transfer phenomena from cylinders compare to that for flow analysis. i.e. [1], [2], [3], [4], [5] etc. Similarly heat transfer from a cylinder was also studied by many authors i.e.[6], [7], [8], [9, 10], [11] but thermal effects on cylinder array were studied less. i.e. [12], [13], [14] and [15]. The work done for thermal analyses around cylinder was done using change in cylinder shape which may be semi circular [7] or circular or square cylinder [11]. From inspection of
available literature it was also clear that bulk of the study includes air as fluid then water was at second choice for fluid. Similarly from Uniform Heat Flux boundary condition and Constant Wall Temperature (CWT) Boundary condition on cylinder, CWT was mainly used in most of the work. The wall effect on cylinder was also studied less.

2. MATHEMATICAL MODEL AND NUMERICAL APPROACH

The case considered here is to numerically analyze 2D viscous unsteady incompressible flow having temperature \( T_{\infty} \) across a heated square array of circular cylinders having temperature \( T_s \) to check wall effect on different thermal parameters. It was assumed that fluid properties do not change with change in temperature and flow was unsteady, two dimensional, viscous and incompressible. At inlet of domain flow was uniform and assumed to start impulsively from the rest; it passes over an infinite long cylinder in Z direction. Here mainly three governing equations have to be solved in steps or together are Continuity, momentum and energy equations.

The schematics of models for single cylinder and square array of circular cylinders is shown in Figure 2.1.

![Figure 2.1 Schematic of Models for (a) single cylinder (b) symmetry, (c) and (d) asymmetry of cylinders in domain](image)

2.1 MATHEMATICAL MODEL

The variables for length, x and y velocity, time, pressure and temperature in governing equations were converted to dimensionless form as:

\[
x = \frac{x}{D}, \quad u = \frac{u}{u_\infty}, \quad v = \frac{v}{v_\infty}, \quad t = \frac{t}{u_\infty \delta}, \quad P = \frac{P}{\rho u_\infty^2}.
\]

\[
T = \frac{\theta - T_s}{T_{\infty} - T_s}
\]

With aforementioned assumptions of fluid, Continuity, momentum and energy equations in Cartesian coordinates can be written as

- Continuity equation:
1. X momentum equation:
\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{Re} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \]

2. Y momentum equation:
\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{Re} \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] \]

3. Energy equation:
\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{Pr} \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \]

The Peclet number, \( Pe = Re \times Pr \)

where \( Re = \frac{\rho DU}{\mu} \) and \( Pr = \frac{c_p}{\lambda} \)

The dimensionless local rate of heat transfer along cylinder surface is given as
\[ N u_{local} = \frac{hL}{k} = -\left( \frac{\partial T}{\partial n} \right) \]

CWT Boundary condition case was applied only in symmetric model of square array of circular cylinders, for this case
\[ N u_{local} = -\frac{\partial T}{\partial n} \]

UHF Boundary condition case was applied in symmetric and asymmetric models of square array of circular cylinders, for this case
\[ N u_{local} = \frac{1}{T} \]

The drag & lift coefficients were calculated as

\[ C_D = \frac{F_L}{0.5 \rho U_m^2 D} \] and \[ C_L = \frac{F_D}{0.5 \rho U_m^2 D} \]

Where \( F_L \) and \( F_D \) are the total lift and drag forces exerted by the fluid acting on cylinder per unit cylinder length. Therefore \( C_D = C_{DP} + C_{DF} \) where \( C_{DP} \) and \( C_{DF} \) are pressure and friction drag coefficients respectively. The frequency of vortex shedding (f) in wake region is given by Strouhal number (St) and it is defined as
\[ St = \frac{fD}{U_m} \]

2.2 NUMERICAL APPROACH

The governing equations were solved using boundary conditions specified below:

- **At the outer left Inlet boundary:** Uniform velocity inlet in x direction i.e.
  \[ u=1, v=0, T=T^\infty \]

- **At the Cylinder surface:** No slip condition was applied i.e.
  \[ u=0, v=0, \frac{\partial T}{\partial n} = 0 \]

- **At outflow boundary:** Constant pressure condition was applied. i.e.
  \[ P=0, u=1, v=0, \frac{\partial T}{\partial n} = 0 \]

- **At upper & lower boundary:**
  - For case of symmetrical cylinders: Neuman boundary condition (symmetry) for velocity and temperature was applied. i.e. \[ \frac{\partial u}{\partial y} = 0, v = 0, \frac{\partial T}{\partial y} = 0, \frac{\partial T}{\partial x} = 0 \]
  - For case of asymmetry of cylinders: Wall with no slip condition was applied i.e. \[ u = 0, v = 0, \frac{\partial T}{\partial y} = 0, \frac{\partial T}{\partial x} = 0 \]

2.3 MODELING OF SQUARE ARRAY MODEL

The meshing in case of symmetrical square array of cylinders is shown in Figure2.2. Here figure 2.2 (a) shows Grid with full domain figure 2.2 (b) shows Zoom view around cylinders and figure 2.2 (c) shows boundary layer for square array of circular cylinders. Meshing for each case was done using quadrilateral multi-block grid.

![Figure 2.2 Meshing in symmetric square array of circular cylinders case](image)

3. RESULTS AND DISCUSSION

The flow and thermal analyses were done on a heated cylinder at various values of Reynolds number i.e. 50, 100 and 200 with Prandtl number kept constant as Pr=0.7. Similar kind of analyses over heated square array of circular cylinders were done for various Reynolds numbers i.e. 40, 50, 100, 150 and 200 by varying different Prandtl number i.e. 0.7, 1, 7, 10 and 20. The analyses were done on square array of cylinders with length between centers of two cylinders to diameter ratio (L/D) was kept constant as
1.25. Effect of two asymmetric arrangements i.e. 5 unit and 10 unit asymmetry of cylinders in domain were analyzed. Change of boundary conditions i.e. UHF and CWT on cylinder array were also analyzed but for only symmetric arrangement of cylinder array in domain at above mentioned Reynolds number and Prandtl number. Hence total 100 numerical analyses were done for square array of circular cylinders. In analyses for square array of cylinders, fully developed velocity field was obtained initially then thermal analyses were done on fully converged field.

3.1 Validation for Single Cylinder

The present analysis of cross flow around a heated cylinder for air was validated for average Nusselt number at Re=150 and Re=200 with available numerical result of [14] and with values obtained from correlations of Zhuaukas, Knudsen et al. and Churchill et al. The comparison is given in Table 3.1.

Table 3.1 Comparison of Average Nusselt number

<table>
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<tbody>
<tr>
<td>200</td>
<td>7.371</td>
<td>7.474</td>
<td>7.21</td>
<td>7.16</td>
<td>7.19</td>
</tr>
</tbody>
</table>

The variation of local Nusselt number along the cylinder was also compared with the results presented by [14] for Re=200. The comparison of same was presented in Figure 3.2.

Figure 3.1 Local Nusselt number at Re=200

It was noted that the present curve behavior was matching with [14], Maximum value of average Nusselt number was nearly same for both curve approximately at 180° but separation from cylinder in present case was earlier compare to that of [14]. The average Nusselt number as well as the local Nusselt number was found in well agreement with the available literature. The correlations used by various authors, with which the present results been compared are given in Table 3.2

Table 3.2 Correlations of previous authors

<table>
<thead>
<tr>
<th>Author</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zhuaukas</td>
<td>$N_u = 0.51 Re^{1/2}$</td>
</tr>
<tr>
<td>Knudsen et al.</td>
<td>$N_u = 0.663 Re^{-0.46} Pr^{1/3}$</td>
</tr>
<tr>
<td>Churchill et al.</td>
<td>$N_u = 0.3 + \frac{0.62 Re^{1/3} Pr^{1/3}}{[1 + (0.4 \frac{Re}{Pr})^{-1}][1 + (\frac{Re}{252000})^{0.63}]}$</td>
</tr>
</tbody>
</table>

3.2 Qualitative result & discussion

3.2.1. Effect on vorticity contour

The Figure 3.2 describes the vorticity generation and Karman vortex street phenomenon for Re=150. The present analysis was done for 200 non-dimensional time. It was observed that the flow got separated from upstream cylinders but no wake formation observed between upstream and downstream cylinders when cross flow over cylinder array took place. Hence it was concluded that all these four cylinders acts as a single bluff body and single Karman vortex street got generated. These wakes at downstream side of cylinder array were symmetric in manner and developing up to 125 dimensionless time which is shown in Figure 3.2 (a) to (d). First time at 125 dimensionless time, the shear layer emerged from lower surface of cylinder was step ahead to that emerged from upper surface which is shown in Figure 3.2 (e). From Figure (f), it can be seen that the vortex of negative vorticity grew in the upper side of the cylinder. As time progressed, this upper vortex gets distorted and cutoff from the upper shear layer and shed into the wake of the cylinder. Then the positive vortex induced in the separated lower shear layer which grows and a maximum positive lift occurs. In such a manner periodic behavior of flow was started happening slowly which can be seen from Figure 3.2 (f) to (j). The flow became fully periodic with constant rate of vortex formation at 162 non-dimensional time which was shown in Figure 3.2 (k) and then flow was repetitive and producing wake at constant interval.
3.2.2. Effect on Temperature Contour

The instantaneous temperature contours for flow passing over cylinder array with UHF boundary condition in symmetric domain having Re=150, Pr=0.7 were plotted in Figure at various non-dimensional time. As both the vorticity and thermal energy were being transported by the flow in the wake, the contours of vorticity and isotherms have some similar features. The temperature near the cylinder surface was higher and decreasing as it elongated in downstream direction. The temperature rise was quick till dimensionless time reached 40. This observation was clear from Figure (a) and (b). Then the rise in temperature gradient was low, hence temperature was not increasing rapidly till 135 dimensionless time which was shown in Figure (c) to (h). After 135 dimensionless time, temperature around cylinder was almost constant.
3.3 Quantitative result & discussion

3.3.1 Effect of Reynolds number and Prandtl No.

The Figure 3.5 shows effect of Re on average Nu at each Pr of study. The Figure shows that with rise in Re, average Nusselt number increases for each Prandtl number. It was also noted down that with increase in Prandtl number from 0.7 to 20, increase in average Nusselt number at low Reynolds number i.e. Re=40 was less compared to increase of same at higher value of Reynolds number i.e. Re=200.

![Figure 3.5 Effect of Re on average Nu at each Pr](image)

4.2.2 Effect of Asymmetry

The Figure 3.7 represents effect of cylinder array asymmetry on average Nusselt number at Re=40. Asymmetry also effects on heat transfer. The Figure shows that with increase in asymmetry of cylinders in domain, average Nusselt number increases. This increase in average Nusselt no. with increase in 5 unit & 10 unit asymmetry of cylinders were 4.69% and 7.17% respectively. The reason for this greater Nu was that due to asymmetry of cylinders in domain, wakes behind cylinders got affected by domain wall. The flow passing near to domain wall helped in generating vortices behind cylinder.

![Figure 3.7 Effect of asymmetry on average Nu no.](image)

4.2.3 Correlation of average Nusselt number

The various correlations for average Nusselt number as functions of Reynolds number and Prandtl number were developed as mentioned in table

<table>
<thead>
<tr>
<th>For condition</th>
<th>Correlation</th>
</tr>
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<tbody>
<tr>
<td>CWT symmetric</td>
<td>$Nu = 0.559Re^{1/2}Pr^{1/3} - 0.257$</td>
</tr>
<tr>
<td>UHF symmetric</td>
<td>$Nu = 0.668Re^{1/2}Pr^{1/3} - 0.26$</td>
</tr>
<tr>
<td>5unit asymmetry</td>
<td>$Nu = 0.697Re^{1/2}Pr^{1/3} - 0.288$</td>
</tr>
<tr>
<td>10unit asymmetry</td>
<td>$Nu = 0.704Re^{1/2}Pr^{1/3} - 0.274$</td>
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5. Conclusion

The following conclusions are drawn from the present study.
• For Single cylinder, average Nusselt number in present case was within 5% with referred literature and peak value of local Nusselt number was approximately similar to validated result.
• Quantitatively, with increase in 5 unit & 10 unit asymmetry of cylinders, increase in average Nusselt no. were 4.69% and 7.17% respectively.
• Average Nusselt no. is 18 to 25% higher in case of UHF boundary condition compare to CWT boundary condition for symmetric domain.
• Average Nusselt number of upstream cylinders was higher compare to counterpart.
• Qualitatively with increase in Pr, temperature variation decreases around cylinders for constant value of Re.
• The correlations developed for average Nusselt number in different conditions were presented in Table 3.3.

6. REFERENCES